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# Assorted Iterative Methods as a Solution of Fuzzy Linear Systems

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## ABSTRACT

In this paper some iterative methods for solving fuzzy linear systems by two crisp linear systems is presented. Also necessary and sufficient conditions for existence of solution are given. Therefore, this paper presents a review on iterative methods as a solution for a general $n \times n$ fuzzy system of linear equations of the form $Ax = b$ . It is hope that this review will provide an input in the study of fuzzy linear systems. The efficiency of the iterative methods is illustrated by offering a numerical example.

**Keywords: Fuzzy linear systems, Fuzzy numbers, Iterative methods.**

## INTRODUCTION

The $n$ x $n$ fuzzy linear system of equations has been studied by many authors. A general model for solving a fuzzy linear system was first proposed by Friedman *et al.* (1998). They used the embedding method and replaced the original fuzzy linear system by a crisp linear system with a nonnegative coefficient matrix S, which may be singular even if A is nonsingular. Asady *et al*. (2005) who merely discuss the full row rank system, use the same method to solve the fuzzy linear system for. Zheng and Wang (2006) discuss the solution of the general consistent and inconsistent fuzzy linear system. Based on some previous studies, fuzzy linear systems are most generally solved by iterative methods Allahviranloo (2004, 2005), Dehgan and Hashemi (2006), Feng (2008), Hadjidimos (1978) and Rakhdimos (2008). The main aim of this paper is to review all iterative methods to solve fuzzy linear systems. This paper is organized in the following way: firstly, some basic definitions on iterative method, fuzzy numbers and fuzzy linear system are given. Then, iterative methods to solve problem of fuzzy linear systems are proposed. Besides, the algorithms which offered by numerical example are illustrated and this paper ends with conclusion.

## PRELIMINARIES

In this section, some necessary backgrounds and notions of iterative method, fuzzy linear systems, and fuzzy numbers are reviewed.

### Iterative Method

Iterative method for solving linear system $Ax = b$ begins with initial guess for solution and successively improves it until solution is as accurate as desired. In theory, infinite number of iterations might be required to converge to exact solution. In practice, iteration terminates when residual $\| b - Ax \|$, or some other measure of error is as small as desired. Iterative methods are especially useful when matrix $A$ sparse because no fill is incurred.

**Fuzzy Linear Systems**

Definitions of fuzzy linear systems have been provided by Allahviranloo (2004, 2005), Allahviranloo and Kermani (2006), Asady *et al.* (2005), Friedman *et al*. (1998), Kandel *et al*. (1996) and Zheng and Wang (2006) as follows:

$$\begin{array}{l} a_{11}x_1 + a_{12}x_2 + .... + a_{1n}x_n = y_1 \\ a_{21}x_1 + a_{22}x_2 + .... + a_{2n}x_n = y_2 \\ \vdots \quad \vdots \qquad \vdots \quad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + .... + a_{nn}x_n = y_n \end{array} \tag{1}$$

The $n$ x $n$ matrix form of the above equations is $AX = Y$, where the coefficient matrix $A = (a_{ij}), 1 \le i,j \le n$ is a crisp $n$ x $n$ matrix and $y_i \in E^1, 1 \le i \le n$. This system is called a fuzzy linear system (FLS).

Fuzzy numbers [1], [2], [5], [7], [12] , [13] and [15]
A fuzzy number is a fuzzy set $u{:}R^1 \to I = [0,1]$ which satisfies

i. $u$ is upper semicontinuous.

ii. *u(x)*= 0 outside some interval,[c, d].

iii. There are real numbers $a, b{:}c \le a \le b \le d$ for which

   a. *u(x)* is monotonic increasing on[c, a].
   b. *u(x)* is monotonic decreasing on [b, d].
   c. *u(x)* $= u(x) = 1, a \le x \le b$.

The set of all the fuzzy numbers is denoted by $E^1$.

## TERATIVE METHODS TO SOLVE FUZZY LINEAR SYSTEMS

**Jacobi Method**

Allahviranloo (2004) introduced the Jacobi method to solve fuzzy linear system for the first time. By this method, he described that without loss of generality, suppose that $s_{ii} > 0$ for all $i = 1, ......., 2n$. Let $S = D + L + U$ where

$$D = \begin{bmatrix} D_1 & 0 \\ 0 & D_1 \end{bmatrix}, L = \begin{bmatrix} L_1 & 0 \\ S_2 & L_1 \end{bmatrix}, U = \begin{bmatrix} U_1 & S_2 \\ 0 & U_1 \end{bmatrix}.$$

$(D_1)_{ii} = s_{ii} 0, i = 1, ...., n$ and suppose $S_1 = D_1 + L_1 + U_1$. In Jacobi method, from the structure of $SX = T$,

$$\begin{bmatrix} D_1 & 0 \\ 0 & D_1 \end{bmatrix} \begin{bmatrix} \underline{X} \\ \overline{X} \end{bmatrix} + \begin{bmatrix} L_1 + U_1 & S_2 \\ S_2 & L_1 + U_1 \end{bmatrix} \begin{bmatrix} \underline{X} \\ \overline{X} \end{bmatrix} = \begin{bmatrix} \underline{Y} \\ \overline{Y} \end{bmatrix} \tag{2}$$

So the Jacobi iterative technique will be

$$\begin{aligned} \underline{X}^{(k+1)} &= D_1^{-1}\underline{Y} - D_1^{-1}(L_1+U_1)\underline{X}^k - D_1^{-1}S_2\overline{X}^k \\ \overline{X}^{(k+1)} &= D_1^{-1}\overline{Y} - D_1^{-1}(L_1+U_1)\overline{X}^k - D_1^{-1}S_2\underline{X}^k \end{aligned} \tag{3}$$

**Gauss Seidel Method**

Allahviranloo(2004) also proposed Gauss Seidel method as one of the iterative methods to solve fuzzy linear system. He points out that in the Gauss Seidel method,

$$\begin{bmatrix} D_1+L_1 & 0 \\ S_2 & D_1+L_1 \end{bmatrix}\begin{bmatrix} \underline{X} \\ \overline{X} \end{bmatrix} + \begin{bmatrix} U_1 & S_2 \\ 0 & U_1 \end{bmatrix}\begin{bmatrix} \underline{X} \\ \overline{X} \end{bmatrix} = \begin{bmatrix} \underline{Y} \\ \overline{Y} \end{bmatrix} \tag{4}$$

Then,

$$\begin{aligned} \underline{X} &= (D_1+L_1)^{-1}\underline{Y} - (D_1+L_1)^{-1}U_1\underline{X} - (D_1+L_1)^{-1}S_2\overline{X} \\ \overline{X} &= (D_1+L_1)^{-1}\overline{Y} - (D_1+L_1)^{-1}U_1\overline{X} - (D_1+L_1)^{-1}S_2\underline{X} \end{aligned} \tag{5}$$

So the Gauss Seidel iterative technique will be

$$\begin{aligned} \underline{X}^{(k+1)} &= (D_1+L_1)^{-1}\underline{Y} - (D_1+L_1)^{-1}U_1\underline{X}^k - (D_1+L_1)^{-1}S_2\overline{X}^k \\ \overline{X}^{(k+1)} &= (D_1+L_1)^{-1}\overline{Y} - (D_1+L_1)^{-1}U_1\overline{X}^k - (D_1+L_1)^{-1}S_2\underline{X}^k \end{aligned} \tag{6}$$

**Successive Over Relaxation (SOR) Method**

Allahviranloo (2005) has recently developed SOR method as one new iterative method in solving fuzzy linear system. He founds that, SOR method is a modification of the Gauss Seidel iteration. By multiply system of (4) in $D^{-1}$,

$$\begin{bmatrix} I+D_1^{-1}L_1 & 0 \\ S_2 & I+D_1^{-1}L_1 \end{bmatrix}\begin{bmatrix} \underline{X} \\ \overline{X} \end{bmatrix} + \begin{bmatrix} D_1^{-1}U_1 & S_2 \\ 0 & D_1^{-1}U_1 \end{bmatrix}\begin{bmatrix} \underline{X} \\ \overline{X} \end{bmatrix} = \begin{bmatrix} D_1^{-1}\underline{Y} \\ D_1^{-1}\overline{Y} \end{bmatrix} \tag{7}$$

Let $D_1^{-1}U_1 = U_1, D_1^{-1}L_1 = L_1$. Then, he finds out that

$$\begin{bmatrix} I+L_1 & 0 \\ S_2 & I+L_1 \end{bmatrix}\begin{bmatrix} \underline{X} \\ \overline{X} \end{bmatrix} + \begin{bmatrix} U_1 & S_2 \\ 0 & U_1 \end{bmatrix}\begin{bmatrix} \underline{X} \\ \overline{X} \end{bmatrix} = \begin{bmatrix} D_1^{-1}\underline{Y} \\ D_1^{-1}\overline{Y} \end{bmatrix} \tag{8}$$

and

$$\begin{aligned} (I+L_1)\underline{X} &= D^{-1}\underline{Y} - U_1\underline{X} - S_2\overline{X} \\ (I+L_1)\overline{X} &= D^{-1}\overline{Y} - U_1\overline{X} - S_2\underline{X} \end{aligned} \tag{9}$$

for some parameter $\omega$.

$$\begin{aligned}(I+\omega L_1)\underline{X} &= \omega D^{-1}\underline{Y} - [(1-\omega)I + \omega U_1]\underline{X} - \omega S_2 \overline{X}\\ (I+\omega L_1)\overline{X} &= \omega D^{-1}\overline{Y} - [(1-\omega)I + \omega U_1]\overline{X} - \omega S_2 \underline{X}\end{aligned} \tag{10}$$

If $\omega = 1$, then clearly is just the Gauss Seidel solution (9). So the SOR iterative method will be

$$\begin{aligned}\underline{X}^{(k+1)} &= (I+\omega L_1)^{-1}\omega D^{-1}\underline{Y} - (I+\omega L_1)^{-1}[(1-\omega)I+\omega U_1]\underline{X}^{(k)} - (I+\omega L_1)^{-1}\omega S_2 \overline{X}^{(k)}\\ \overline{X}^{(k+1)} &= (I+\omega L_1)^{-1}\omega D^{-1}\overline{Y} - (I+\omega L_1)^{-1}[(1-\omega)I+\omega U_1]\overline{X}^{(k)} - (I+\omega L_1)^{-1}\omega S_2 \underline{X}^{(k)}\end{aligned} \tag{11}$$

Based on these three methods, Allahviranloo (2004, 2005) points out that Jacobi, Gauss Seidel and SOR method iterates are converge to the unique solution $X = A^{-1}Y$, for any $X^0$, where $X \in R^{2n}$ and $(\underline{X}, \overline{X}) \in E^n$. The stopping criterion with tolerance is $t > 0$ is

$$\frac{\| \underline{X}^{k+1} - \underline{X}^{k} \|}{\| \underline{X}^{k+1} \|}\varepsilon, \frac{\| \overline{X}^{k+1} - \overline{X}^{k} \|}{\| \overline{X}^{k+1} \|}\varepsilon, k = 0, 1, \ldots\ldots$$

**Richardson Method**

Dehgan *et al.* (2006) proposed Richardson method as iterative method to solve fuzzy linear system. In this method, $Q$ is chosen to be the identity matrix of order $2n$. Then, $X = (I - S)X + Y$ and the Richardson iteration matrix is

$$Q_{RICH} = \begin{bmatrix} I_n - B & -C \\ -C & I_n - B \end{bmatrix}.$$

Hence, the Richardson method will be in the following form:

$$\begin{aligned}\underline{X}^{(k+1)} &= \underline{Y} + (I_n - B)\underline{X}^{(k)} + C\overline{X}^{(k)}\\ \overline{X}^{(k+1)} &= \overline{Y} + (I_n - B)\overline{X}^{(k)} + C\underline{X}^{(k)}\end{aligned} \tag{12}$$

In this approach, they found that the sequence $\{X^m\}$ is easily computed, but the rate of convergence of the sequence $\{X^m\}$ is very slow. By this method, $\rho(Q_{RICH}) = \max\{|1 - m(S)|, |1 - M(S)|\}$. So when $S$ is a symmetric positive definite matrix, then the Richardson method converges if and only if $M(S) < 2$.

**The Extrapolated Richardson Method**

Dehgan *et al*. (2006) performed the extrapolated Richardson method to show the new one iterative method to solve fuzzy linear systems. In the study, they found that $Q = \frac{1}{\alpha} I_{2n}$, where $\alpha > 0$ is called the extrapolation parameter. So, in this case, $X = (I - \alpha S)X + \alpha Y$ and the extrapolated Richardson iteration matrix as

$$Q_{ER} = \begin{bmatrix} I_n - \alpha B & -\alpha C \\ -\alpha C & I_n - \alpha B \end{bmatrix}.$$

Besides, they write the extrapolated Richardson method in the following forms:

$$\begin{aligned}\underline{X}^{(m+1)} &= \alpha\underline{Y} + (I_n - \alpha B)\underline{X}^{(m)} + \alpha C\overline{X}^{(m)} \\ \overline{X}^{(m+1)} &= \alpha\overline{Y} + (I_n - \alpha B)\overline{X}^{(m)} + \alpha C\underline{X}^{(m)}\end{aligned} \tag{13}$$

**Jacobi Over Relaxation (JOR) Method**

Based on Dehgan *et al.* (2006), Jacobi over Relaxation method is an extrapolated of Jacobi method.

$$\begin{bmatrix}\underline{X} \\ \overline{X}\end{bmatrix} = \begin{bmatrix}\omega D_1^{-1}\underline{Y} + (I_n - \omega D_1^{-1}B)\underline{X} + \omega D_1^{-1}C\overline{X} \\ \omega D_1^{-1}\overline{Y} + (I_n - \omega D_1^{-1}B)\overline{X} + \omega D_1^{-1}C\underline{X}\end{bmatrix}$$

So the JOR iterative method will be

$$\begin{cases}\underline{X^{(m+1)}} = \omega D_1^{-1}\underline{Y} - \omega D_1^{-1}\left[\left(1 - \frac{1}{\omega}\right)D_1 + L_1 + U_1\right]\underline{X^{(m)}} + \omega D_1^{-1}C\overline{X^{(m)}} \\ \overline{X^{(m+1)}} = \omega D_1^{-1}\overline{Y} - \omega D_1^{-1}\left[\left(1 - \frac{1}{\omega}\right)D_1 + L_1 + U_1\right]\overline{X^{(m)}} + \omega D_1^{-1}C\underline{X^{(m)}}\end{cases}$$

Hence we have the JOR method in matrix form:

$$X^{(m+1)} = D^{-1}(D - \omega S)X^{(m)} + \omega D^{-1}Y \tag{14}$$

Evidently the JOR method is reduced to Jacobi method for $\omega = 1$.

**The Extrapolated Gauss Seidel Method**

Dehgan *et al.*(2006) introduced the forward extrapolated Gauss Seidel method with

$$Q = \frac{1}{\alpha}(D + L) = \frac{1}{\alpha}\begin{bmatrix}D_1 + L_1 & 0 \\ C & D_1 + L_1\end{bmatrix}$$

where $\alpha$ is the extrapolation parameter. So the extrapolated Gauss Seidel iteration matrix is

$$Q_{EGS} = \begin{bmatrix}\gamma_1 & \gamma_2 \\ \gamma_3 & \gamma_4\end{bmatrix}$$

For $\alpha = 1$, the extrapolated Gauss Seidel method coincides with the Gauss Seidel method, and the extrapolated Gauss Seidel method is

$$\begin{aligned}\underline{X}^{(m+1)} &= \alpha(D_1 + L_1)^{-1}\underline{Y} + (D_1 + L_1)^{-1}[(1-\alpha)D_1 + (1-\alpha)L_1 - \alpha U_1]\underline{X}^{(m)} \\ &\quad + \alpha(D_1 + L_1)^{-1}C\overline{X}^{(m)} \\ \overline{X}^{(m+1)} &= \alpha(D_1 + L_1)^{-1}C(D_1 + L_1)^{-1}\underline{Y} + \alpha(D_1 + L_1)^{-1}\overline{Y} + \\ &\quad (D_1 + L_1)^{-1}\left[(1-\alpha)D_1 + (1-\alpha)L_1 - \alpha U_1 + \alpha C(D_1 + L_1)^{-1}C\right]\overline{X}^{(m)} \\ &\quad - \alpha(D_1 + L_1)^{-1}C(D_1 + L_1)^{-1}U_1\underline{X}^{(m)}\end{aligned} \tag{15}$$

**The Accelerated Over Relaxation (AOR) Method**

The AOR method is first introduced by Hadjidimos [11]. Then, Dehgan *et al.* [8] used this method for solving a fuzzy linear system by using two parameters. There are relaxation parameter $(\alpha)$ and extrapolation parameter. $(\omega)$.
For forward AOR method,

$$Q = \frac{1}{\omega}(D + \alpha L) = \frac{1}{\omega}\begin{bmatrix} D_1 + \alpha L_1 & 0 \\ \alpha C & D_1 + \alpha L_1 \end{bmatrix}.$$

So, $Q^{-1} = \omega\begin{bmatrix} (D_1 + \alpha L_1)^{-1} & 0 \\ -\alpha(D_1 + \alpha L_1)^{-1}C(D_1 + \alpha L_1)^{-1} & (D_1 + \alpha L_1)^{-1} \end{bmatrix}$ and $Q_{\alpha,\omega} = \begin{bmatrix} \gamma_1 & \gamma_2 \\ \gamma_3 & \gamma_4 \end{bmatrix}$.

Hence, the AOR method is

$$\begin{aligned} \underline{X}^{(m+1)} &= \omega(D_1 + \alpha L_1)^{-1}\underline{Y} + (D_1 + \alpha L_1)^{-1}[(1-\omega)D_1 + (\alpha - \omega)L_1 - \omega U_1]\underline{X}^{(m)} \\ &\quad - \omega(D_1 + \alpha L_1)^{-1}C\overline{X}^{(m)} \\ \overline{X}^{(m+1)} &= -\alpha\omega(D_1 + \alpha L_1)^{-1}\underline{Y} + \omega(D_1 + \alpha L_1)^{-1}\overline{X}^{(m)} + (D_1 + \alpha L_1)^{-1} \\ &\quad [(1-\omega)D_1 + (\alpha - \omega)L_1 - \omega U_1 + \alpha\omega C(D_1 + \alpha L_1)^{-1}C]\overline{X}^{(m)} \\ &\quad - \omega(D_1 + \alpha L_1)^{-1}C(D_1 + \alpha L_1)^{-1}[(1-\alpha)D_1 - \alpha U_1]\underline{X}^{(m)} \end{aligned} \tag{16}$$

**Symmetric SOR Method**

Dehgan *et al.* (2006) proposed the following formula:

$$Q = \frac{1}{\omega(2-\omega)}(D + \omega L)D^{-1}(D + \omega U).$$

When this formula used, the symmetric SOR iteration matrix will be obtained.

**Unsymmetric SOR Method**

The Unsymmetric SOR method differs from the symmetric SOR method in the second SOR part each iteration where a different relaxation factor is used. Dehgan *et al.*, (2006) proposed the following formula:

$$Q = \frac{1}{\omega_1 + \omega_2 - \omega_1\omega_2}(D + \omega_1 L)D^{-1}(D + \omega_2 U).$$

**Extrapolated Modified Aitken Method**

The extrapolated modified Aitken iterative method was first introduced by Evans, (1963) as a method for solving the systems of linear algebraic equations arising from discretizing of the elliptic difference equations. This method could be easily used for solving the fuzzy linear systems by taking

$$Q = \frac{1}{\omega}(D + \omega L)D^{-1}(D + \omega U).$$

## NUMERICAL RESULT

To illustrate the proposed iterative methods, a numerical example is given based on the previous researches.

This example reproduced by Tofigh Allahviranloo on 2004.
We consider 3 x 3 fuzzy system.

$$4x_1 + x_2 - x_3 = (r, 2 - r)$$
$$-x_1 + 3x_2 + x_3 = (2 + r, 3)$$
$$2x_1 + x_2 + 3x_3 = (-2, -1 - r)$$

This numerical example will be solved by some of iterative methods as mention above. They are Jacobi, Gauss Seidel, Richardson, Successive over Relaxation, Jacobi over Relaxation and Accelerated over Relaxation methods.

llahviranloo (2004) produced the following exact solution:
The elements of solution vector are as follows:

$$x_1 = (0.1399 * r - 0.4125, 0.3217 * r - 0.0351)$$
$$x_2 = (0.2894 * r + 0.9125, -0.0970 * r - 1.1076)$$
$$x_3 = (-0.1897 * r - 0.6969, 0.1513 * r + 0.7353)$$

By the fact that $x_3$ is not a fuzzy number. The fuzzy solution in this case is a weak solution given by

$$x_1 = (0.1399 * r - 0.4125, 0.3217 * r - 0.0351)$$
$$x_2 = (0.2894 * r + 0.9125, -0.0970 * r - 1.1076)$$
$$x_3 = (-0.1513 * r - 0.7353, 0.1897 * r + 0.6969)$$

However, the approximation solution by using Jacobi method is as follows:

$$x_1 = (0.140 * r - 0.413, 0.322 * r - 0.049)$$
$$x_2 = (0.289 * r + 0.915, -0.097 * r - 1.107)$$
$$x_3 = (-0.190 * r - 0.696, 0.151 * r + 0.735)$$

The approximation solution obtained by Gauss Seidel method is

$$x_1 = (0.1399 * r - 0.4126, 0.3217 * r - 0.0490)$$
$$x_2 = (0.2893 * r + 0.9152, -0.0970 * r - 1.1075)$$
$$x_3 = (-0.1897 * r - 0.6967, 0.1512 * r + 0.7351)$$

Hence by using Successive over Relaxation method, the following approximation solution is produced.

$$x_1 = (0.140 * r - 0.412, 0.321 * r - 0.049)$$
$$x_2 = (0.289 * r + 0.915, -0.097 * r - 1.108)$$
$$x_3 = (-0.190 * r - 0.696, 0.151 * r + 0.734)$$

The approximation solution by Jacobi over Relaxation method is as follows:

$$x_1 = (0.140 * r - 0.412, 0.322 * r - 0.049)$$
$$x_2 = (0.289 * r + 0.915, - 0.097 * r - 1.107)$$
$$x_3 = (- 0.190 * r - 0.696, 0.151 * r + 0.735)$$

Then, by using Accelerated over Relaxation method, the following approximation solution is produced.

$$x_1 = (0.173 * r - 0.139, 0.098 * r - 0.212)$$
$$x_2 = (0.381 * r + 0.886, - 0.051 * r - 0.233)$$
$$x_3 = (- 0.242 * r - 0.8690.072 * r + 0.329)$$

However, Richardson method not produced any approximation solution because it divergent to the system. Therefore, in order to illustrate the whole approximation solution produced by these iterative methods, a triangular fuzzy numbers will be given for all approximation solutions produced.

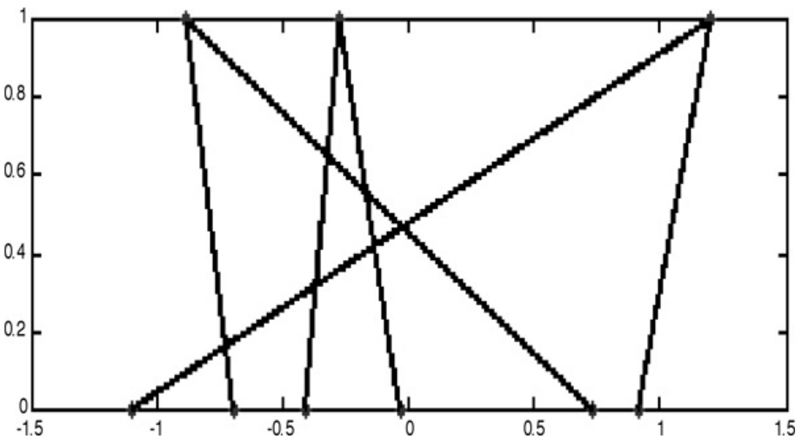


*Figure 1: Triangular fuzzy number for exact solution*

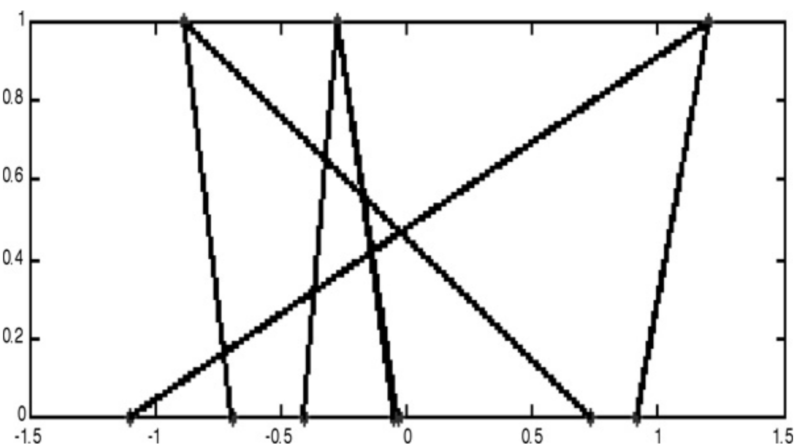


*Figure 2: Approximation solution by Jacobi method*

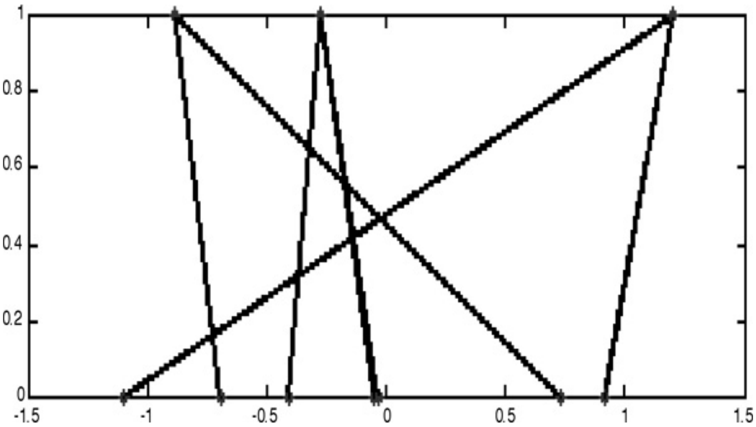


*Figure 3: Approximation solution by Gauss Seidel method*

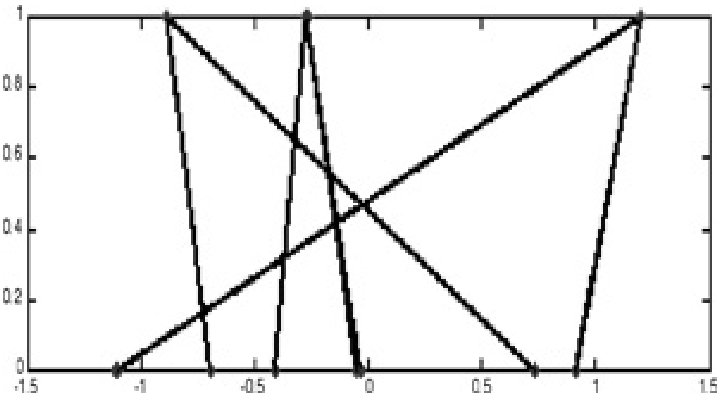


*Figure 4: Approximation solution by Successive over Relaxation method*

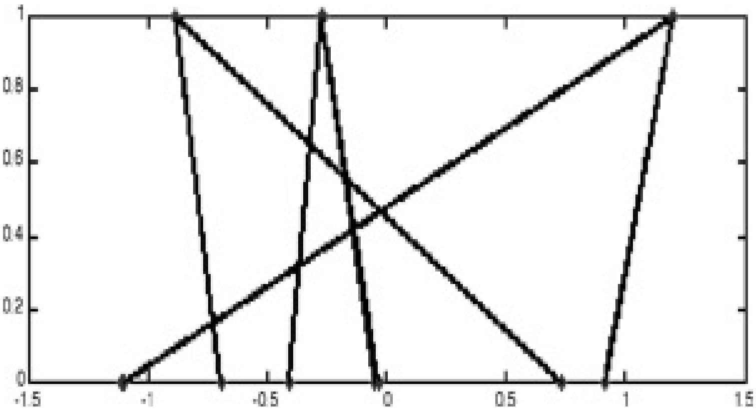


*Figure 5: Approximation solution by Jacobi over Relaxation method*

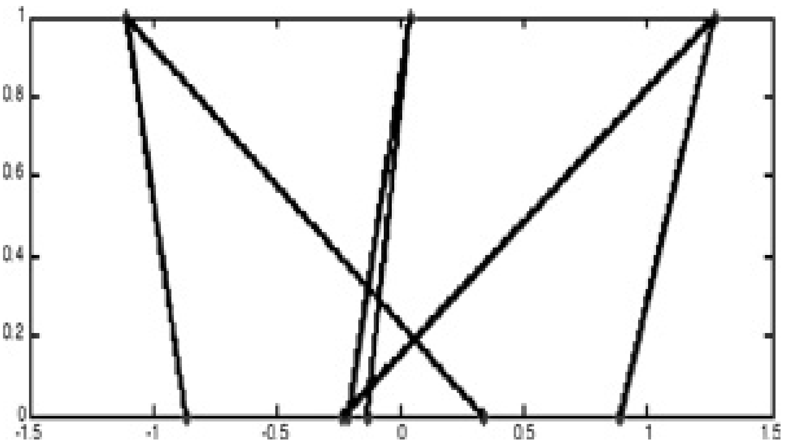


*Figure 6: Approximation solution by Accelerated over Relaxation method*

Hence, we also summarized the number of required iterations for each method converges to the solution in the following table:

**Table 1**: Number of iteration and computer time taking to converge

| Method | Number of iteration | Computer time |
|---|---|---|
| Jacobi | 19 | 0.10 seconds |
| Gauss Seidel | 13 | 0.09 seconds |
| Successive over Relaxation | 14 | 0.09 seconds |
| Jacobi over Relaxation | 18 | 2.93 seconds |
| Accelerated over Relaxation | 14 | 2.42 seconds |
| Richardson | Divergent | - |

## CONCLUSION

In this paper, some iterative methods have been presented to solve fuzzy linear systems. They are Jacobi, Gauss Seidel, Successive over Relaxation, Jacobi over Relaxation, Accelerated over Relaxation, Richardson and a few more iterative methods. A practical example was studied, a 3 x 3 fuzzy linear system. The analysis of results shows that Jacobi method takes the most number of iteration, 19 iterations to converge, as compared to other method, within the same tolerance factor. Besides, it also takes 0.10 seconds for computer time. However according on table 1, Gauss Seidel takes the smallest number of iterations to converge to the solution. It's computing time is similar with Successive over Relaxation method, 0.09 seconds.

Even though it takes the same number of iteration for the two other methods to converge, but computer time is differ, as that of the Successive over Relaxation method, takes 0.09 seconds while Accelerated over Relaxation method takes 2.42 seconds. This shows that Successive over Relaxation requires less computer storage than the Accelerated over Relaxation method. One of the more significant findings to emerge from this study is that iterative methods are properly works for fuzzy linear systems. Therefore these findings enhance our understanding about several numerical techniques in order to determine solution of fuzzy linear systems.

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